Fractional Linear (Möbius) Transformations

Definition and Derivative

Lecture Notes

October 15, 2026

1 Definition

Definition (Möbius map)

Let $a, b, c, d \in \mathbb{C}$ with $ad - bc \neq 0$ $\left(= \det \begin{pmatrix} a & b \\ c & d \end{pmatrix} \right)$. The map

$$z \mapsto \frac{az + b}{cz + d}$$

is called a **fractional-linear map** (or **Möbius map**) – a rational map of order 1.

Its derivative,

$$\left(\frac{az + b}{cz + d} \right)' = \frac{ad - bc}{(cz + d)^2},$$

exists (and is nonzero) wherever $z \neq -d/c, \infty$.